

Inference at \* 1 1  
of proof for Lemma wellfounded\_functionality\_wrt\_iff:

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1.  $T_1 : \text{Type}$ 
2.  $T_2 : \text{Type}$ 
3.  $r_1 : T_1 \rightarrow T_1 \rightarrow \mathbb{P}$ 
4.  $r_2 : T_2 \rightarrow T_2 \rightarrow \mathbb{P}$ 
5.  $T_1 = T_2$ 
6.  $\forall x, y:T_1. r_1(x,y) \iff r_2(x,y)$ 
 $\vdash \text{WellFnd}\{i\}(T_1;x,y.r_1(x,y)) \Rightarrow \text{WellFnd}\{i\}(T_2;x,y.r_2(x,y))$ 
  by ((BackThruLemma 'wellfounded_functionality_wrt_implies')
    CollapseTHEN (
      (Auto_aux (first_nat 1:n) ((first_nat 1:n),(first_nat 3:n)) (first_tok :t) inil_term)))·

1:

 $\vdash T_1 = T_2$ 
2:

 $\vdash \forall x, y:T_1. \{r_1(x,y) \Leftarrow r_2(x,y)\}$ 
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